P425/1
PURE MATHEMATICS
Paper 1

ALL SAINTS S.S
HOME LEARNING TEST

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** the ***eight*** questions in Section **A** and **five** questions from Section **B**.

Any additional question(s) answered will **not** be marked.

All working **must** be shown clearly.

Begin each answer on a **fresh** sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40 MARKS)

Turn Over

1. Solve the simultaneous equations

$$xy = 2$$
$$2\log(x-1) = \log y$$

(05 marks)

2. Differentiate $y = \frac{x-2}{\sqrt{(1-x^2)}}$ with respect to x. *(05 marks)*

3. Without using tables or calculators, show that $\tan^2 22.5^0 = 3 - 2\sqrt{2}$.

(05 marks)

4. Find the gradients of the two tangents from the point $(3, -2)$ to the circle $x^2 + y^2 = 4$. *(05 marks)*

5. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin x}{3 + 5\cos x}\, dx$. *(05 marks)*

6. The points A and B have position vectors $4\mathbf{i} + 3\mathbf{j}$ and $\mathbf{i} + t\mathbf{j}$. Determine the values of t such that the angle $A\hat{O}B = \cos^{-1}\frac{2}{\sqrt{5}}$, where O is the origin. *(05 marks)*

7. A hemispherical bowl of internal radius 15 cm contains water to a depth of 7 cm, find the volume of the water in the bowl correct to 1 decimal place. *(05 marks)*

8. Compute the sum of four- digit numbers formed with the four digits 2, 5, 3, 8 if each digit is used only once in each arrangement.

(05 marks)

SECTION B: (60 MARKS)

Answer **only** five questions. All questions carry equal marks.

9. (a) Show that the curve $x = 5 - 6y + y^2$ represents a parabola and find the directrix. *(05 marks)*

(b) (i) Find the equation of the chord through the points

$P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ of the parabola $y^2 = 4ax$.

(ii) Show that the chord in (b) (i) cuts the directrix where

$$y = \frac{2a(pq-1)}{p+q}$$

(07 marks)

10. (a) The roots of the equation $2x^2 - 3x + 5 = 0$ are α and β. Find the equation whose roots are $\frac{\alpha}{\beta-2}$ and $\frac{\beta}{\alpha-2}$. *(06 marks)*

(b) Solve the equation $\sqrt{\frac{x-1}{3x+2}} + 2\sqrt{\frac{3x+2}{x-1}} = 3$. *(06 marks)*

11. (a) Given that $x = \sec A - \tan A$, prove that $\tan\frac{A}{2} = \frac{1-x}{1+x}$. *(05 marks)*

(b) Solve the equation $\sin t\cos 3t + \sin 3t\cos t = 0.8$ for $0 \le t \le 2\pi$.

(05 marks)

12. Given that $y = \sin(2\sin^{-1}x)$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$. Hence, by Maclaurin's theorem, expand y as far as the term in x^3.

(12 marks)

13. The position vectors of points P and Q are $2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and $3\mathbf{i} - 7\mathbf{j} + 12\mathbf{k}$ respectively.

(a) Determine the length of $\mathbf{PQ}$. *(03 marks)*

(b) Given that the line PQ meets the plane $4x + 5y - 2z = 5$ at the point T. Find the:

(i) co-ordinates of T,

(ii) angle between line PQ and the plane. *(09 marks)*

Turn Over

14. (a) Express $y = \frac{x^4 + 2x}{(x-1)(x^2+1)}$ into partial fractions. *07 marks)*

(b) Evaluate $\int_2^4 y\,dx$. *(05 marks)*

15. (a) Show that $2-3i$ is a root to the equation.

$z^4 - 5z^3 + 18z^2 - 17z + 13 = 0$.

Hence find the other roots of the equation. *(06 marks)*

(b) Using De Moivre's theorem, find the cube roots of $-4+6i$.

(06 marks)

16. (a) Solve the differential equation $\frac{dr}{d\theta} + 2r\tan\theta = \sec^2\theta$. *(05 marks)*

(b) The tangent at any point $P(x, y)$ on the curve, cuts the x-axis at A and the y-axis at B. Given that $2\mathbf{AP} = \mathbf{PB}$ and that the curve passes through the point $(1, 1)$, find the equation of the curve. *(07 marks)*

END